

Weakly chained matrices, policy iteration, and impulse control

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May 12, 2016

Abstract

This work is motivated by numerical solutions to Hamilton-Jacobi-Bellman quasivariational inequalities (HJBQVIs) associated with combined stochastic and impulse control problems. A direct control scheme for such an HJBQVI takes the form of a Bellman problem (BP) involving an operator which is not necessarily contractive. We consider the well-posedness of the BP and give sufficient conditions for convergence of a policy iteration to its unique solution. In the event that these conditions do not hold, weaker conditions guarantee uniqueness, from which it is possible to salvage the convergence of policy iteration by (roughly speaking) pruning policies that render the operator appearing in the BP singular. These results are established using weakly chained diagonally dominant matrices, which give a graph-theoretic characterization of weakly diagonally dominant M-matrices. The BP also happens to be the dynamic programming equation associated with an infinite-horizon Markov decision process with vanishing discount factor (a generalization of reflecting boundaries), which is of independent interest. This work appears in Azimzadeh, P., and P. A. Forsyth. "Weakly Chained Matrices, Policy Iteration, and Impulse Control." *SIAM Journal on Numerical Analysis* 54.3 (2016): 1341-1364.