

## Abstract

Biofilms are microbial depositions on the solid surface exposed to an aqueous solution. Bacteria attach to the substratum and start producing Extracellular Polymeric Substances (EPS), in which they are themselves embedded. The EPS matrix offers resistance against eradication and this makes the biofilm different with suspended planktonic community. The beneficial properties of biofilm have been used in Environmental Engineering for many years, originally in waste water treatment. On the other hand, bacterial biofilms can have detrimental effects especially in the food industry and in medicine. Several mathematical models including individual based and continuum models have been introduced recently to model the biofilm formation. We consider fully continuum model which is highly non-linear degenerate diffusion PDE. It inherits two non-linear diffusion effects. (i) porous medium degeneracy that leads to a finite speed of biofilm/liquid interface propagation and (ii) super-diffusion singularity that assures the solutions should be bounded. The degeneracy introduces gradient blow up which in numerics can lead to interface smearing. The singularity can cause arithmetical problems and force the numerical method to work with very small time-step. Also, the absence of Lipschitz condition for spatial discrete system of considered PDE model due to singularity makes it difficult to use the standard arguments for mathematical analysis. We use the regularisation idea to analyze the degenerate semi-discrete problem and show that it has positive bounded solution. Then we explore time-adaptive method with error control ability to optimize the time-step size. The challenge in applying time-adaptive methods such as Rosenbrock-Wanner method specially for big size coupled problems in term of the number of equations with non-linear reaction terms is computing the Jacobian matrix analytically. We suggest numerical scheme and computer algebra based derivation as alternatives to find the Jacobian matrix. The results of comparing their CPU-time will be presented. Besides the mathematical analysis we also conduct two computational convergence tests. The numerical method is validated in a simulation study.