

The Fibonacci-Mandelbrot Polynomials and Matrices

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Abstract:

We explore a family of polynomials similar to the Mandelbrot polynomials called the Fibonacci-Mandelbrot polynomials defined by $q_0(z) = 0$, $q_1(z) = 1$, and $q_{n+1}(z) = zq_n(z)q_{n-1}(z) + 1$. We compute the roots of the Fibonacci-Mandelbrot polynomials using two methods. One method uses a recursively constructed matrix, where elements are 0, 1, or -1 , whose eigenvalues are the roots of $q_n(z)$. The other method uses homotopy continuation method, where the solution of the differential equation, $\frac{dz}{dt} = \frac{-1}{q'_n(z)}$, in which the initial condition is 0, and the roots of q_{n-1} and q_{n-2} , are also the roots of the Fibonacci-Mandelbrot polynomials.